

Quantum-mechanical Landau-Lifshitz equation

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Quantum-mechanical analogue of Landau-Lifshitz equation has been derived. It has been established that Landau-Lifshitz equation is fundamental physical equation underlying the dynamics of spectroscopic transitions and transitional phenomena. New phenomenon is predicted: electrical spin wave resonance (ESWR) being to be electrical analogue of magnetic spin wave resonance.

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Correct formal description of dynamics of spectroscopic transitions as well as a number of transitional effects, that is, for instance, Rabi-oscillations, free induction and spin echo effects in magnetic resonance spectroscopy and their optical analogues in optical spectroscopy is achieved in the frame of a gyroscopic model, see, e.g., [1, 2]. Mathematical base for gyroscopic model is Landau-Lifshitz (L-L) equation. L-L equation was postulated by Landau and Lifshitz for macroscopic classical description of the motion of magnetization vector in ferromagnets as early as 1935 [3]. L-L equation is as follows:

$$\frac{d\vec{S}}{dt} = [\gamma_H \vec{H} \times \vec{S}], \quad (1)$$

where \vec{S} is magnetic moment, \vec{H} is effective magnetic field, γ_H is gyromagnetic ratio. L-L equation was substantiated quantum-mechanically in magnetic resonance theory, but as the equation, describing the only the motion of magnetic moment in external magnetic field. At the same time L-L equation and Bloch equations, which are based on L-L equations, were in fact postulated for description of *dynamics* of magnetic resonance *transitions*, see, e.g., [4]. The gyroscopic model for optical transition dynamics and for description of optical transitional effects was introduced formally on the base of analogy with gyroscopic model, developed for magnetic resonance. However, the optical analogue of L-L equation was obtained quantum-statistically by means of density operator formalism [1, 2]:

$$\frac{d\vec{P}}{dt} = [\vec{P} \times \gamma_E \vec{E}], \quad (2)$$

where \vec{P} , \vec{E} are vectors, which are considered to be some mathematical abstractions, since their components represent themselves various physical quantities. So, in [1] is emphasized that the only P_x , P_y and E_x , E_y components of vectors \vec{P} , \vec{E} , correspondingly, characterize the genuine electromagnetic properties of the system, at the same time the components P_z and E_z cannot be referred to electromagnetic characteristics. Further, γ_E was called gyroelectric ratio the only tentatively, its analogy with gyromagnetic ratio was suggested. The formal character of optical gyroscopic model is indicated also in modern quantum optics theory, see, e.g., [2]. This situation

is consequence of the attempt to preserve the only polar symmetry properties for the vectors of electric field strength and electric polarization of the medium. At the same time the vectors \vec{P} , \vec{E} in eq.2 should be axial vectors. To built the axial vectors, satysfying eq.2, the third components of polar vectors of electric field strength and electric polarization is changed artificially into quantities which are believed to be some mathematical abstractions in the sence, that they cannot be referred to electric field strength and electric polarization correspondingly. So, it is generally accepted, that the components of the vectors \vec{P} , \vec{E} in eq.2 are consisting from various physical quantities. This situation seems to be incorrect, if to proceed from the assumption of the identity of the nature of the spectroscopic transitions in optical and radiospectroscopy regions.

The aim of given work is to obtain quantum-mechanical equations for description of dynamics of both magnetic resonance and optical transitions with clear physical sence of all the quantities (for the case of simple 1D model of quantum system).

Let us consider the general properties of electromagnetic field to clarify the sence of the quantities in (2). It is well known, that electromagnetic field can be characterized by both contravariant tensor $F^{\mu\nu}$ (or covariant $F_{\mu\nu}$) and contravariant pseudotensor $\tilde{F}^{\mu\nu}$ which is dual to $F^{\mu\nu}$ (or covariant $\tilde{F}_{\mu\nu}$, which is dual to $F_{\mu\nu}$). For instance, $\tilde{F}^{\mu\nu}$ is determined by the following relation: $\tilde{F}^{\mu\nu} = \frac{1}{2}e^{\alpha\beta\mu\nu}F_{\alpha\beta}$, where $e^{\alpha\beta\mu\nu}$ is Levi-Chivita fully antisymmetric unit 4-tensor. The use of field tensors and pseudotensors seems to be equally possible by description of electromagnetic field and its interaction with a matter. So, for example, by using of both field tensor and pseudotensor the field invariants are obtained [5, 6]. Further in the practice of treatment of experimental results is generally accepted, that electric field strength, dipole moment and polarization vectors are polar vectors but magnetic field strength, dipole moment and magnetization vectors are always axial vectors. However for the effects, describing the interaction of electromagnetic field in optical experiments the picture seems to be reverse. It follows directly from the structure of algebraic linear space, which produce field of tensors and pseudotensors.

Really, if the structure of $F^{\mu\nu}$ is

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -H_3 & H_2 \\ E_2 & H_3 & 0 & -H_1 \\ E_3 & -H_2 & H_1 & 0 \end{bmatrix} \quad (3)$$

then the structure of $\tilde{F}_{\mu\nu}$ will be:

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & -H_1 & -H_2 & -H_3 \\ H_1 & 0 & E_3 & -E_2 \\ H_2 & -E_3 & 0 & E_1 \\ H_3 & E_2 & -E_1 & 0 \end{bmatrix}. \quad (4)$$

So we have for contravariant and covariant electromagnetic field pseudotensors the expressions:

$$\tilde{F}^{\mu\nu} = (-\vec{H}, \vec{E}), \quad \tilde{F}_{\mu\nu} = (\vec{H}, \vec{E}). \quad (5)$$

We consider further formally the same vectors \vec{H} and \vec{E} , that is, the vectors consisting of the components, determined by the structure of $\tilde{F}^{\mu\nu}$, but now vector \vec{H} is polar and vector \vec{E} is axial. The possibility of given consideration requires the validation, which can be obtained from the following. Let us define the space F of sets of contravariant tensors $\{F_{\mu\nu}\}$ and pseudotensors $\{\tilde{F}_{\mu\nu}\}$ and corresponding to them sets of covariant tensors and pseudotensors of electromagnetic field over the field of scalar values P . It is evident that all the axioms of linear space hold true, i.e., if $F_1^{\mu\nu}$ and $F_2^{\mu\nu} \in F$, then

$$F_1^{\mu\nu} + F_2^{\mu\nu} = F_3^{\mu\nu} \in F, \quad (6)$$

and, if $F^{\mu\nu} \in F$, then

$$\alpha F^{\mu\nu} \in F \quad (7)$$

for $\forall \alpha \in P$. Let us define the linear algebra \mathfrak{F} by means of definition in above defined vector space $\langle F, +, \cdot \rangle$ of transfer operation $(*)$ to dual tensor, using the convolution with Levi-Chivita fully antisymmetric unit 4-tensor $e_{\alpha\beta\mu\nu}$. It is evident, that in algebra $\langle \mathfrak{F}, +, \cdot, * \rangle$ the following axioms of linear algebra hold true: if $F_1^{\alpha\beta}$ and $F_2^{\alpha\beta} \in \mathfrak{F}$, then

$$e_{\mu\nu\alpha\beta}(F_1^{\alpha\beta} + F_2^{\alpha\beta}) = (\tilde{F}_1)_{\mu\nu} + (\tilde{F}_2)_{\mu\nu} \in \mathfrak{F}, \quad (8)$$

$$(F_1^{\alpha\beta} + F_2^{\alpha\beta})e_{\alpha\beta\mu\nu} = F_1^{\alpha\beta}e_{\alpha\beta\mu\nu} + F_2^{\alpha\beta}e_{\alpha\beta\mu\nu} \in \mathfrak{F}, \quad (9)$$

$$(e_{\alpha\beta\mu\nu}\lambda F^{\mu\nu}) = \lambda(e_{\alpha\beta\mu\nu}F^{\mu\nu}) = (e_{\alpha\beta\mu\nu}F^{\mu\nu})\lambda \quad (10)$$

for $\forall \lambda \in P$. We can determine now on the space $\langle F, +, \cdot \rangle$ the functional Φ as follows:

$$\Phi(F^{\mu\nu}) \equiv \langle F^{\mu\nu} | \Phi \rangle = F^{\mu\nu} \tilde{F}_{\mu\nu}, \quad (11)$$

and

$$\Phi(\tilde{F}^{\mu\nu}) \equiv \langle \tilde{F}^{\mu\nu} | \Phi \rangle = \tilde{F}^{\mu\nu} F_{\mu\nu}. \quad (12)$$

In other words, the mapping $\Phi : F^{\mu\nu}, \tilde{F}^{\mu\nu} \rightarrow \tilde{P}$ is taking place, where \tilde{P} is pseudoscalar field. It is clear, that Φ on the space F is linear functional. Really, let $\alpha, \beta \in P$, then, taking into account the properties (8) to (10) of $\langle \mathfrak{F}, +, \cdot, * \rangle$, we have

$$\langle F^{\mu\nu} | \alpha\Phi_1 + \beta\Phi_2 \rangle = \alpha \langle F^{\mu\nu} | \Phi_1 \rangle + \beta \langle F^{\mu\nu} | \Phi_2 \rangle, \quad (13)$$

$$\langle \tilde{F}^{\mu\nu} | \alpha\Phi_1 + \beta\Phi_2 \rangle = \alpha \langle \tilde{F}^{\mu\nu} | \Phi_1 \rangle + \beta \langle \tilde{F}^{\mu\nu} | \Phi_2 \rangle. \quad (14)$$

Consequently, the set $\Phi(F^{\mu\nu}, \tilde{F}^{\mu\nu})$ of linear functionals on the space $\langle F, +, \cdot \rangle$ represents itself linear space over the field P (since it is evident, that the conditions like to (6) and (7) hold true), which is dual to space $\langle F, +, \cdot \rangle$. Therefore, we have

$$\langle \Phi, +, \cdot \rangle = \langle F^\times, +, \cdot \rangle. \quad (15)$$

It is substantially, that F^\times is not self-dual. Actually, the "vector" (in algebraic sense), which can be built on basis "vectors" of space F with the projections, which are functionals, corresponding, in accordance with relationships (11), (12), to given basis "vectors" of F , cannot belong to F . Really, its components are pseudoscalars and, being to be considered as coefficients in linear combination of the elements of vector space F , cannot belong to field P (properties (6) (7) do not hold true). At the same time full physical description of dynamical system requires to take into consideration both the spaces (more strictly, Gelfand three should be considered, if corresponding topology is determined in these spaces). On other hand, the description with help of Gelfand three will be equivalent to the following description. We define starting space F over the $P + \tilde{P}$. Then resulting functional space will be self-dual. In extended by such a way space we can choose 4 physically different subspaces: two subspaces 1) $\{F^{\mu\nu}\}$ and 2) $\{\tilde{F}^{\mu\nu}\}$ over the scalar field P and two subspaces 3) $\{F^{\mu\nu}\}$ and 4) $\{\tilde{F}^{\mu\nu}\}$ over the pseudoscalar field \tilde{P} . The second case differs from the first case by the following. Symmetry properties of \vec{E} and \vec{H} remain the same, i.e., \vec{E} is polar vector, since it is dual vector to antisymmetric 3D pseudotensor, and \vec{H} , respectively, is axial. At the same time, the components of vector \vec{E} correspond now to pure space components of field tensor $\tilde{F}^{\mu\nu}$, the components of vector \vec{H} correspond to time-space mixed components. Arbitrary element of the third subspace

$$\alpha F^{\mu\nu}(x_1) + \beta F^{\mu\nu}(x_2), \quad (16)$$

where $\alpha, \beta \in \tilde{P}$, x_1, x_2 are the points of Minkowski space, represents itself the 4-pseudotensor. Its space components, being to be the components of antisymmetric 3-pseudotensor, determine dual polar vector \vec{H} , mixed components are the components of 3-pseudovector \vec{E} . Therefore, the symmetry properties of the components

of vectors \vec{E} and \vec{H} relatively the improper rotations will be inverse to the case 1. It is evident, that in the 4-th case the symmetry properties of the components of vectors \vec{E} and \vec{H} relatively the improper rotations will be inverse to the case 2. Given consideration allows to suggest, that free electromagnetic field is 4-fold degenerated. The interaction with device (or, generally, with some substance) relieves degeneracy and leads to formation of resonance state: field + device (substance), which has finite lifetime. It is reasonable to suggest, that the field in the resonance state becomes nondegenerate. Realization of concrete field state (one of 4 possible) will, evidently, be determined by symmetry characteristics of registering device (interacting substance). So, we suggest, that, in principle, various symmetry properties of the same field can be obtained by registration of interaction with the same substance, but with various methods, e.g., with ESR and optical absorption.

We will consider the case 4 in more detail. The components of \vec{H} and \vec{E} are determined in this case by other potentials, accordingly, by dual scalar $\tilde{\varphi}$ and dual vector $\vec{\tilde{A}}$ potentials, which can be obtained from the solution of the following set of differential equations:

$$\begin{aligned} \nabla \tilde{\varphi}(\vec{r}, t) &= -[\nabla \times \vec{\tilde{A}}(\vec{r}, t)], \\ [\nabla \times \vec{\tilde{A}}(\vec{r}, t)] &= \frac{\partial \vec{\tilde{A}}(\vec{r}, t)}{\partial x^0} - \frac{\partial A_0(\vec{r}, t)}{\partial \vec{r}}, \end{aligned} \quad (17)$$

where $A_0(\vec{r}, t) \equiv \varphi(\vec{r}, t)$ is scalar potential and $\vec{\tilde{A}}(\vec{r}, t)$ is vector potential, which determine the components of genuine field tensors $F^{\mu\nu}$, $F_{\mu\nu}$. It can be shown, that the solution of eq.17 is viewed as follows:

$$\tilde{\varphi} = - \int_0^{\vec{r}} [\nabla \times \vec{\tilde{A}}] d\vec{r}, \quad \vec{\tilde{A}} = \vec{\tilde{A}}_1 + \vec{\tilde{A}}_2, \quad (18)$$

where

$$\vec{\tilde{A}}_1 = -\frac{1}{4\pi} \nabla \left(\int_{(\vec{\rho})} \frac{Q}{|\vec{r} - \vec{\rho}|} d^3\rho \right) \Rightarrow \vec{\tilde{A}}_1 = \frac{Q}{4\pi} \frac{\vec{r}}{|\vec{r}|^2}, \quad (19)$$

$$\vec{\tilde{A}}_2 = \left[\nabla \times \frac{1}{4\pi} \int_{(\vec{\rho})} \frac{\frac{\partial \vec{\tilde{A}}}{\partial x^0} - \frac{\partial A_0}{\partial \vec{\rho}}}{|\vec{r} - \vec{\rho}|} d^3\rho \right]. \quad (20)$$

It is taken into account by derivation of expressions (19) and (20), that dual vector potential satisfies to calibration condition: $\nabla \cdot \vec{\tilde{A}}(\vec{r}, t) = Q$, where Q is const. It is evident, that in the case of usually used Coulomb calibration dual vector potential will be determined the only by relationship (20). Thus the simple analysis shows, that, if electric field components are components of *pseudotensor*, the equation of dynamics of *optical* transitions will have mathematically the same structure with that one for

the equation of dynamics of *magnetic resonance* transitions (in which magnetic field components are parts of genuine tensor $F^{\mu\nu}$). In other words, mathematical abstractions in eq.2 become real physical sense: \vec{E} is vortex part of intracrystalline and external electric field, \vec{P} is electrical moment, which should be defined like to magnetic moment. So, both the vectors are axial vectors, that is, they have necessary symmetry properties (relatively reflection and inversion), in order to satisfy the eq.2. It should be noted that the statement on "equality in rights" of genuine field tensor and pseudotensor by description of electromagnetic phenomena follows from general consideration of the *geometry* of Minkowski space, which determine unambiguously the full set of 3 possible kinds of geometrical objects. Really, the tensors and pseudotensors are equally possible geometrical objects among them for any pseudo-Euclidean abstract space, to which Minkowski space is isomorphic [7].

Quantum-mechanical description of dynamics of spectroscopic transitions on the example of 1D system interacting with electromagnetic field confirm given general conclusion. Really, let us consider the system representing itself the periodical ferroelectrically (ferromagnetically) ordered chain of n equivalent elements, interacting with external oscillating electromagnetic field. It is assumed that the interaction between elements (elementary units) of the chain can be described by the Hamiltonian of quantum XYZ Heisenberg model in the case of a chain of magnetic dipoles and by corresponding optical analogue of given Heisenberg model in the case of a chain of electric dipoles. We will consider for the simplicity the case of isotropic exchange. Each elementary unit of the chain will be considered as two-level system like to one-electron atom. Then Hamiltonians for the chain of electrical dipole moments and for the chain of magnetic dipole moments will be mathematically equivalent. We will use further the rotating wave approximation [2]. Then the chain for distinctness of electrical dipole moments can be described by the following Hamiltonian:

$$\begin{aligned} \hat{\mathcal{H}} &= \frac{\hbar\omega_0}{2} \sum_n \hat{\sigma}_n^z - p_E^{\alpha\beta} \sum_n E_1^n (\hat{\sigma}_n^+ e^{-i\omega t} + \hat{\sigma}_n^- e^{i\omega t}) \\ &+ \sum_n [J_E (\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \hat{\sigma}_n^- \hat{\sigma}_{n+1}^+ + \frac{1}{2} \hat{\sigma}_n^z \hat{\sigma}_{n+1}^z) + H.c.]. \end{aligned} \quad (21)$$

where $\hat{\sigma}_n^z = |\alpha_n\rangle \langle \alpha_n| - |\beta_n\rangle \langle \beta_n|$ is so called $\hat{\sigma}_z$ -operator, observable quantity for which is population difference of the states of n -th element, $\hat{\sigma}_n^+ = |\alpha_n\rangle \langle \beta_n|$ and $\hat{\sigma}_n^- = |\beta_n\rangle \langle \alpha_n|$ are transition operators of n -th element from eigenstate $|\alpha_n\rangle$ to eigenstate $|\beta_n\rangle$ and vice versa, correspondingly. It is suggested in the model, that $|\alpha_n\rangle$ and $|\beta_n\rangle$ are eigenstates, producing the full set for each of n elements. It is evident, that given assumption can be realized strictly the only by the absence of the interaction between the elements. At the same time proposed model will rather well describe the real case, if the interaction energy of adjacent elements is much less of the energy of the splitting $\hbar\omega_0 = \mathcal{E}_\beta - \mathcal{E}_\alpha$ between the energy

levels, corresponding to the states $|\alpha_n\rangle$ and $|\beta_n\rangle$. This case includes in fact all known experimental situations. Further, $p_E^{\alpha\beta}$ is matrix element of dipole transitions between the states $|\alpha_n\rangle$ and $|\beta_n\rangle$, which along with energy difference of these states $\mathcal{E}_\beta - \mathcal{E}_\alpha$ are suggested to be independent on n , E_1^n is amplitude of electric component of electromagnetic wave on the n -th element site, \hbar is Planck's constant, J_E is optical analogue of the exchange interaction constant. Here, in correspondence with the suggestion, $J_E = J_E^x = J_E^y = J_E^z$. In the case of the chain of magnetic dipole moments E_1^n in Hamiltonian (21) is replaced by H_1^n , i.e., by amplitude of magnetic component of electromagnetic wave on the n -th element site, J_E is replaced by the exchange interaction constant J_H , matrix element $p_E^{\alpha\beta}$ is replaced by $p_H^{\alpha\beta}$ and the frequency ω_0 is replaced by $\frac{1}{\hbar}g_H\beta_H H_0 = \gamma_H H_0$, where H_0 is external static magnetic field, β_H is Bohr magneton, g_H is g -tensor, which is assumed for the simplicity to be isotropic. The first term in Hamiltonian (21) characterizes the total energy of all chain elements in the absence of external field and in the absence of interaction between chain elements. The second item characterizes an interaction of a chain with an external oscillating electromagnetic field in dipole approximation. Matrix elements of dipole transitions $p_E^{\alpha\beta}$ and $p_E^{\beta\alpha}$ between couples of the states $(|\alpha_n\rangle, |\beta_n\rangle)$ and $(|\beta_n\rangle, |\alpha_n\rangle)$, respectively, are assumed to be equal, i.e., spontaneous emission is not taken into consideration. The third item is, in essence, Hamiltonian of quantum Heisenberg XXX-model in the case of magnetic version and its electrical analogue in electric version of the model proposed. Let us define the vector operator:

$$\hat{\vec{\sigma}}_k = \hat{\sigma}_k^- \vec{e}_+ + \hat{\sigma}_k^+ \vec{e}_- + \hat{\sigma}_k^z \vec{e}_z. \quad (22)$$

It seems to be the most substantial for the subsequent analysis, that a set of $\hat{\sigma}_k^m$ operators, where m is $z, +, -$, produces algebra, which is isomorphic to $S = 1/2$ Pauli matrix algebra, i.e., mappings $f_k : \hat{\sigma}_k^m \rightarrow \sigma_P^m$ realize isomorphism. Here k is a number of chain unit, σ_P^m is the set of Pauli matrices for the spin of $1/2$. Consequently, from physical point of view $\hat{\vec{\sigma}}_k$ represents itself some vector operator, which is proportional to operator of the spin of k -th chain unit. Vector operators $\hat{\sigma}_k^m$ produce also linear space over complex field, which can be called transition space. It is 3-dimensional (in the case of two-level systems), that is 3 operator equations of the motion for components of $\hat{\vec{\sigma}}_k$ is necessary for correct description of optical transitions. Typical inaccuracy in many of the quantum optics calculations of transition dynamics is connected with obliteration of correct dimensionality of transition space. The equation of the motion for $\hat{\vec{\sigma}}_k$ is:

$$i\hbar \frac{\partial \hat{\vec{\sigma}}_k}{\partial t} = [\hat{\sigma}_k^-, \hat{\mathcal{H}}] \vec{e}_+ + [\hat{\sigma}_k^+, \hat{\mathcal{H}}] \vec{e}_- + [\hat{\sigma}_k^z, \hat{\mathcal{H}}] \vec{e}_z. \quad (23)$$

We will consider the case of homogeneous excitation of the chain, that is E_1^n is independent on unit number n ($E_1^n \equiv E_1$). Then, using the commutation rela-

tions $[\hat{\sigma}_k^+, \hat{\sigma}_n^-] = \delta_{kn}\hat{\sigma}_n^z$, $[\hat{\sigma}_n^-, \hat{\sigma}_k^z] = 2\delta_{kn}\hat{\sigma}_n^-$, $[\hat{\sigma}_k^z, \hat{\sigma}_n^+] = 2\delta_{kn}\hat{\sigma}_n^+$, we obtain

$$\frac{\partial \hat{\sigma}_k^z}{\partial t} = 2i\Omega_E (e^{-i\omega t}\hat{\sigma}_k^+ - e^{i\omega t}\hat{\sigma}_k^-) \quad (24a)$$

$$+ \frac{2iJ_E}{\hbar} (\{\hat{\sigma}_k^-, (\hat{\sigma}_{k+1}^+ + \hat{\sigma}_{k-1}^+)\} - \{\hat{\sigma}_k^+, (\hat{\sigma}_{k+1}^- + \hat{\sigma}_{k-1}^-)\}),$$

$$\frac{\partial \hat{\sigma}_k^+}{\partial t} = i\omega_0\hat{\sigma}_k^+ + i\Omega_E e^{i\omega t}\hat{\sigma}_k^z \quad (24b)$$

$$+ \frac{iJ_E}{\hbar} (\{\hat{\sigma}_k^+, (\hat{\sigma}_{k+1}^z + \hat{\sigma}_{k-1}^z)\} - \{\hat{\sigma}_k^z, (\hat{\sigma}_{k+1}^+ + \hat{\sigma}_{k-1}^+)\}),$$

$$\frac{\partial \hat{\sigma}_k^-}{\partial t} = -i\omega_0\hat{\sigma}_k^- - i\Omega_E e^{-i\omega t}\hat{\sigma}_k^z \quad (24c)$$

$$- \frac{iJ_E}{\hbar} (\{\hat{\sigma}_k^-, (\hat{\sigma}_{k+1}^z + \hat{\sigma}_{k-1}^z)\} + \{\hat{\sigma}_k^z, (\hat{\sigma}_{k+1}^- + \hat{\sigma}_{k-1}^-)\}),$$

where expressions in braces $\{\cdot, \cdot\}$ are anticommutants. Here Ω_E and γ_E is Rabi frequency and gyroelectric ratio. They are determined, correspondingly, by relations $\Omega_E = \frac{E_1 p_E^{\alpha\beta}}{\hbar} = \gamma_E E_1$, $\gamma_E = \frac{p_E^{\alpha\beta}}{\hbar}$, which are replaced by relations $\Omega_H = \frac{H_1 p_H^{\alpha\beta}}{\hbar} = \gamma_H H_1$, $\gamma_H = \frac{p_H^{\alpha\beta}}{\hbar}$ in the case of the chain of magnetic dipole moments. The equations (24) can be represented in compact vector form, at that the most simple expression is obtained by using of the basis $\vec{e}_+ = (\vec{e}_x + i\vec{e}_y)$, $\vec{e}_- = (\vec{e}_x - i\vec{e}_y)$, \vec{e}_z . So, we have

$$\frac{\partial \hat{\vec{\sigma}}_k}{\partial t} = [\hat{\vec{\sigma}}_k \times \hat{\vec{\mathfrak{G}}}_{k-1,k+1}], \quad (25)$$

where $\hat{\vec{\sigma}}_k$ is given by (22), but with the components, corresponding to new basis, $k = \overline{2, N-1}$, and vector operator $\hat{\vec{\mathfrak{G}}}_{k-1,k+1} = \hat{\mathfrak{G}}_{k-1,k+1}^- \vec{e}_+ + \hat{\mathfrak{G}}_{k-1,k+1}^+ \vec{e}_- + \hat{\mathfrak{G}}_{k-1,k+1}^z \vec{e}_z$, where its components are

$$\hat{\mathfrak{G}}_{k-1,k+1}^- = \Omega_E e^{-i\omega t} - \frac{2J_E}{\hbar} (\hat{\sigma}_{k+1}^- + \hat{\sigma}_{k-1}^-), \quad (26a)$$

$$\hat{\mathfrak{G}}_{k-1,k+1}^+ = \Omega_E e^{i\omega t} - \frac{2J_E}{\hbar} (\hat{\sigma}_{k+1}^+ + \hat{\sigma}_{k-1}^+), \quad (26b)$$

$$\hat{\mathfrak{G}}_{k-1,k+1}^z = -\omega_0 - \frac{2J_E}{\hbar} (\hat{\sigma}_{k+1}^z + \hat{\sigma}_{k-1}^z). \quad (26c)$$

It should be noted, that vector product of vector operators in (25) is calculated in the correspondence with known expression

$$\left[\hat{\vec{\sigma}}_k \times \hat{\vec{\mathfrak{G}}}_{k-1,k+1} \right] = \begin{vmatrix} \vec{e}_- \times \vec{e}_z & \hat{\sigma}_k^- & \hat{\mathfrak{G}}_{k-1,k+1}^- \\ \vec{e}_z \times \vec{e}_+ & \hat{\sigma}_k^+ & \hat{\mathfrak{G}}_{k-1,k+1}^+ \\ \vec{e}_+ \times \vec{e}_- & \hat{\sigma}_k^z & \hat{\mathfrak{G}}_{k-1,k+1}^z \end{vmatrix}, \quad (27)$$

however, by its calculation one should take anticommutants of corresponding components instead their products. Given definition seems to be natural generalization of vector product for the case of operator vectors, since the only in this case the result will be independent on a sequence of components of both the vectors in their products like to that one for usual vectors. Naturally, the expressions like to (27) can be used for calculation

of vector product of common vectors. Taking into account the physical sense of vector operators $\hat{\sigma}_k$ we conclude, that (25) represent themselves required quantum-mechanical difference-differential equations (the time is varied continuously, the coordinates are varied discretely) for the description of the dynamics of the spectroscopic transitions (in the frames of the model proposed). From here in view of isomorphism of algebras of operators $\hat{\sigma}_k$ and components of the spin it follows that the (25) is equivalent to L-L equation (in its difference-differential form). Consequently we have proved the possibility to use L-L equation for the description of the dynamics of spectroscopic transitions, as well as for the description of transitional effects. To obtain the continuous approximation of (25) for coordinate variables too, we have to suggest that the length of electromagnetic wave λ satisfies the relation: $\lambda \gg a$, where a is 1D-chain-lattice constant. Then the continuous limit is realized if to substitute all the operators, which depend on discrete variable k , for the operators depending on continuous variable z , that is: $\hat{\sigma}_k^\pm \rightarrow \hat{\sigma}^\pm(z)$, $\hat{\sigma}_k^z \rightarrow \hat{\sigma}^z(z)$. Thus, we obtain, taking also into account the relations $\hat{\sigma}_{k+1}^{z,\pm} + \hat{\sigma}_{k-1}^{z,\pm} - 2\hat{\sigma}_k^{z,\pm} \rightarrow a^2 \frac{\partial^2 \hat{\sigma}^{z,\pm}(z)}{\partial z^2}$, the equation, which, like to (25), in compact vector form is:

$$\frac{\partial \hat{\sigma}(z)}{\partial t} = \left[\hat{\sigma}(z) \times \gamma_E \vec{E} \right] - \frac{2a^2 J_E}{\hbar} \left[\hat{\sigma}(z) \times \nabla^2 \hat{\sigma}(z) \right], \quad (28)$$

where $\vec{E} = E_1 e^{i\omega t} \vec{e}_- + E_1 e^{-i\omega t} \vec{e}_+ + \left(\frac{-\omega_0}{\gamma_E} \right) \vec{e}_z$. The structure of vector \vec{E} clarifies its physical meaning. Two components E^+ , E^- are right- and left-rotatory components of oscillating external electric field, third component E^z is intracrystalline electric field, which produces two level energy splitting for each of the unit of a chain system with value, equal to $\hbar\omega_0$. It means that the following relation is taking place: $\omega_0 = \frac{1}{\hbar} g_E \beta_E E_0 = \gamma_E E_0$, where β_E is electrical analogue of Bohr magneton, g_E is electrical analogue of magnetic g -tensor, which is assumed for the simplicity to be isotropic. In other words, by means of given relation the correspondence between an unknown intracrystalline electric field E_0 and observed frequency ω_0 is set up. Further, we take into consideration, that physical sense of operators $\hat{\sigma}(z)$ in continuous limit is remained, i.e., for each point of z the components $\hat{\sigma}^\pm(z)$, $\hat{\sigma}^z(z)$ are satisfying to algebra, which is isomorphic to algebra of the set of spin components. Then by means of relation, which has mathematically the same form for both the types of the systems studied $\hat{S}(z) \sim \frac{\hbar \hat{\sigma}(z)}{2}$, the equation of the motion for operators of magnetic and electric spin moments are obtained. So, e.g., the equation of the motion for electric spin moment operator is:

$$\frac{\partial \hat{S}(z)}{\partial t} = \left[\hat{S}(z) \times \gamma_E \vec{E} \right] - \frac{4a^2 J_E}{\hbar^2} \left[\hat{S}(z) \times \nabla^2 \hat{S}(z) \right] \quad (29)$$

Equation (29) gives for the case $J = 0$ quantum-mechanical optical analogue of classical Landau-Lifshitz

equation in continuous limit, in fact it is operator equation, which argues the correctness of eq.(2), that is, the physical correctness of gyroscopic model for description of optical transitions and transitional optical analogues of magnetic resonance phenomena. If $J \neq 0$ we have quantum-mechanical optical analogue of classical L-L equation, which was introduced by Kittel for SWR description [8]. Therefore, the results obtained allow to predict a new phenomenon - electric spin wave resonance (ESWR). The equation (29) (if put aside the operator symbols) and equation introduced by Kittel are coinciding mathematically to factor 2 in the second term. This difference is like to well known difference of gyromagnetic ratios for orbital and spin angular moments. Thus along with magnetic resonance methods we can detect a spin value of particles, quasiparticles, impurities or other centers in solids by optical methods: by study of transitional optical analogues of magnetic resonance phenomena or ESWR. It should also be noted that above considered theoretical description of ESWR allow predict the difference in splitting constants which characterize ESWR by its experimental detection with using of one-photon methods like to IR-absorption or IR-reflection and with using of two-photon methods like to Raman scattering. It is evident that equations (28), (29) can immediately be used for single transition methods, for instance, for IR-absorption. By Raman scattering we have two subsequent transitions. Then operator $\hat{\sigma}(z)$, which characterizes the transition dynamics by Raman scattering process, has to be consisting from two components $\hat{\sigma}_1(z)$ and $\hat{\sigma}_2(z)$ characterizing both the transitions, taken separately, that is $\hat{\sigma}(z) = \hat{\sigma}_1(z) + \hat{\sigma}_2(z)$. Consequently, the equation for transition dynamics of second component, which will determine experimentally observed ESWR-spectrum, is

$$\frac{\partial \hat{\sigma}_2(z)}{\partial t} = \left[\hat{\sigma}_2(z) \times \gamma_E \vec{E} \right] - \frac{2a^2 J}{\hbar} \left[\hat{\sigma}_2(z) \times \nabla^2 \hat{\sigma}_2(z) \right] - \frac{2a^2 J}{\hbar} \left[\hat{\sigma}_2(z) \times \nabla^2 \hat{\sigma}_1(z) \right]. \quad (30)$$

The second and the third items in eq.19 are practically equal to each other, since we are dealing with interacting electric dipole moments of the same chain, that is, $\nabla^2 \hat{\sigma}_1(z)$ and $\nabla^2 \hat{\sigma}_2(z)$ have almost equal values. Then we obtain, that the value of splitting constant by Raman scattering detection of ESWR in the same sample is almost double in comparison with that one by IR detection of ESWR. The observation of doubling in the splitting constant by Raman ESWR-studies is additional direct argument in ESWR identification.

Therefore, quantum-mechanical analogue of Landau-Lifshitz equation has been derived with clear physical sense of the quantities for both radio- and optical spectroscopy. It has been established that Landau-Lifshitz equation is fundamental physical equation underlying the dynamics of spectroscopic transitions and transitional phenomena. New phenomenon - electrical spin wave res-

onance and its main properties are predicted.

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